A special case of the log-sum inequality most relevant here is the following.

\[
\iint p(y_1, y_2) \log \frac{p(y_1, y_2)}{q(y_1, y_2)} dy_1 dy_2 \geq \iint p(y_1, y_2) dy_2 \log \frac{p(y_1, y_2)}{q(y_1, y_2)} dy_2 dy_1 \\
\geq \int p(y_1) \log \frac{p(y_1)}{q(y_1)} dy_1
\]

Consequently, \( D_{y_1, y_2}(\alpha_1 \parallel \alpha_2) \geq D_{y_1}(\alpha_1 \parallel \alpha_2) \), which means the KL distance increases with more in the population. Here, I have suppressed the input since it does not affect the result. This says that regardless of the population’s structure, increasing the population size can increase KL. What is interesting about the non-cooperative case is that its special form allows us to say that the KL \textit{strictly} increases.