The US 2000–2002 market descent: how much longer and deeper?

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Abstract

A remarkable similarity in the behaviour of the US S&P500 index from 1996 to August 2002 and of the Japanese Nikkei index from 1985 to 1992 (11 year shift) is presented, with particular emphasis on the structure of the bearish phases. Extending a previous analysis of Johansen and Sornette on the Nikkei index ‘antibubble’ based on a theory of cooperative herding and imitation working both in bullish as well as in bearish regimes, we demonstrate the existence of a clear signature of herding in the decay of the S&P500 index since August 2000 with high statistical significance, in the form of strong log-periodic components. In the next two years, we predict an overall continuation of the bearish phase, punctuated by local rallies; we predict an overall increasing market until the end of the year 2002 or until the first quarter of 2003; we predict a severe following descent (with maybe one or two severe ups and downs in the middle) which stops during the first semester of 2004. Beyond this, we cannot be very certain due to the possible effect of additional nonlinear collective effects and of a real departure from the antibubble regime. The similarities between the two stock market indices may reflect deeper similarities between the fundamentals of the two economies which both went through over-valuation with strong speculative phases preceding the transition to bearish phases characterized by a surprising number of bad surprises (bad loans for Japan and accounting frauds for the US) sapping investors’ confidence.

1. Introduction

Financial bubbles have been proposed to be critical phenomena in the sense of the statistical physics of critical phase transitions (see [13, 14, 21, 28, 31, 32] and references therein). Two hallmarks of criticality have been documented: i) super-exponential power-law acceleration of prices towards a ‘critical’ time $t_c$ corresponding to the end of the speculative bubble and ii) log-periodic modulations accelerating according to a geometric series signalling a discrete hierarchy of timescales.

Imitation between investors and their herding behaviour not only lead to speculative bubbles with accelerating overvaluations of financial markets possibly followed by crashes, but also to ‘antibubbles’ with decelerating market devaluations following market peaks. There is thus a certain
degree of symmetry between the speculative behaviour of the ‘bull’ and ‘bear’ market regimes. This degree of symmetry, after the critical time $t_c$ corresponds to the existence of ‘antibubbles’, characterized by a power-law decrease of the price (or of the logarithm of the price) as a function of time $t > t_c$, down from a maximum at $t_c$ (which is the beginning of the antibubble) and by decelerating/expanding log-periodic oscillations [15]. The classic example of such an antibubble is the long-term depression of the Japanese index, the Nikkei, that has decreased along a downward path marked by a succession of ups and downs since its all-time high of 31 December 1989. Another good example is found in the gold future prices after 1980, after their all-time high. The Russian market prior to and after its speculative peak in 1997 also constitutes a remarkable example where both bubble and antibubble structures appear simultaneously for the same $t_c$. This is, however, a rather rare occurrence, probably because accelerating markets with log-periodicity often end up in a crash, a market rupture that thus breaks down the symmetry ($t_c - t$ for $t < t_c$ into $t - t_c$ for $t > t_c$). Herding behaviour can occur and progressively weaken from a maximum in ‘bearish’ (decreasing) market phases, even if the preceding ‘bullish’ phase ending at $t_c$ was not characterized by an irritable run-away. The symmetry is thus statistical or global in general and holds in the ensemble rather than for each single case individually.

In [15, 16], the decrease of the Nikkei index has been analysed in detail, starting from 1 January 1990, using three increasingly complex formulae, corresponding to the three successive orders of a Landau expansion around 0 of the logarithm of the price: $\frac{d \ln f(t)}{dt} = \alpha + \cdots$, where in general the coefficients may be complex. The first-order expression of stock indices had been derived by using the symmetry of discrete scale invariance [26], according to which the price times series is invariant (around a critical time) with respect to magnifications that are integer powers of a fundamental scaling ratio. This leads to

$$\ln p(t) \approx A_1 + B_1 t^\alpha + C_1 t^\alpha \cos[\omega \ln(t) + \phi_1],$$

(1)

with

$$\tau = t - t_c,$$

(2)

where $t_c$ is the time of the beginning of the antibubble. The inclusion of a nonlinear quadratic term in the Landau expansion leads to the second-order log-periodic formula [30]

$$\ln p(t) \approx A_2 + \frac{\tau^\alpha}{\sqrt{1 + \left(\frac{t}{\Delta_t}\right)^{2\alpha}}} \left[ B_2 + C_2 \cos \left[ \omega \ln(t) + \phi_2 \right] + \Delta_\omega \frac{\Delta_\omega}{2\alpha} \ln \left(1 + \left(\frac{t}{\Delta_t}\right)^{2\alpha}\right) \right].$$

(3)

A third-order formula has also been given in [15] which derives from the addition of a third-order term in the Landau expansion. We do not give this formula explicitly here as we shall not need it for the present analysis. Equation (3) describes a transition from an angular log-frequency $\omega$ (for $\tau < \Delta_t$) to a different angular log-frequency $\omega + \Delta_\omega$ (for $\Delta_t < \tau$).

Note that expression (3) reduces to equation (1) in the limit $\Delta_t \to +\infty$. Using these three formulae, a prediction was published in January 1999 on the behaviour of the Japanese stock market in the following two years [15], that has been remarkably successful [16].

The present situation of the USA after the burst of the ‘new economy’ bubble in March–April 2000 [17] does not seem different anymore from that of Japan, paralleling the end of the Japanese bubble in January 1990, and the cascade of discoveries, not yet fully unveiled to their full extent, of the creative accounting of companies striving to look good in the eyes of analysts rather than to build strong fundamentals (paralleling the discovery of a surprising amount of bad loans.
held by Japanese banks). This remark takes more force by looking at figure 1, which compares the behaviour of the Japanese Nikkei index and the US S&P500 index with a time shift of 11 years. The three fits of the Nikkei index, shown in figure 1 as undulating curves, use the three mathematical expressions discussed above. The dashed curve is the simple log-periodic formula (1); the dotted curve is the improved nonlinear log-periodic formula (3) developed in [30] and also used for the October 1929 and October 1987 crashes with 8 years of data; the continuous line is the extension of the previous nonlinear log-periodic formula to the third-order Landau expansion developed in [15]. This last more sophisticated mathematical formula predicts the transition from an angular log-frequency ω (for τ < Δτ) to another angular log-frequency ω + Δω for Δτ < τ < Δτ′ and to a third angular log-frequency ω + Δω + Δω′ (for Δτ′ < τ).

In the next section, we use the insight provided by the theory of critical herding [13, 14, 21, 28, 31, 32] to analyse the S&P500 2000–2002 antibubble. We perform a battery of tests, starting with parametric fits of the index with two of the above log-periodic power-law formulae, followed by the application of the Shank transformation to characteristic times. We then present two spectral analyses, the Lomb periodogram applied to the parametrically detrended index and the non-parametric (H, q)-analysis of fractal signals. These different approaches complement each other and confirm the remarkably strong presence of log-periodic structures. We also detect a significant second-order harmonic which provides a statistically significant improvement of the description of the data by the theory, as tested using the statistical theory of nested hypotheses. Section 3 offers an analysis of the future evolution of the S&P500 index over the next two years by comparing the predictions of three formulae. These predictions are found to be robust and consistent. We conclude by speculating in section 4 on possible consequences as well as projections even further ahead.


2.1. Theoretical foundations

The analysis presented below relies on a general theory of financial crashes and of stock market instabilities developed in a series of works (see [13, 14, 21, 28, 31, 32] and references therein). The main ingredient of the theory is the existence of positive feedbacks which has been documented to occur in stock markets [4] as well as in the economy [1]. Positive feedback, i.e., self-reinforcement, refers to the fact that financial agents who herd tend to imitate other agents more strongly in the future if herding has led to success in the past. Translated into market prices, conditioned on the observation that the market has recently moved up (respectively down), this makes it more probable for it to keep moving up (respectively down), so that a large cumulative move may ensue. 'Positive feedback' is the opposite of 'negative feedback', a concept well-known for instance in population dynamics: the larger the population of rabbits in a valley, the less grass per rabbit. If the population grows too much, the rabbits will eventually starve, slowing down their reproduction rate and thus reducing their population at a later time. Thus negative feedback means that the higher the population, the slower the growth rate, leading to a spontaneous regulation of the population size; negative feedback thus tends to regulate growth towards an equilibrium. In contrast, positive feedback asserts that the higher (respectively lower) the price or the price return in the recent past, the higher (respectively lower) will be the price growth in the future. Positive feedback, when unchecked, can produce runaways until the deviation from equilibrium is so large that other effects (such as the decrease of liquidity, adverse news on credit, on taxes and on fundamentals) can have an abrupt effect and lead to ruptures or crashes. Alternatively, it can give prolonged depressive bearish markets.

There are many mechanisms leading to positive feedbacks including derivative hedging, insurance portfolios, investors’ over-confidence, imitative behaviour and herding between investors. Such positive feedbacks provide the fuel for the development of speculative bubbles as well as antibubbles [11, 15, 16], by the mechanism of cooperativity, that is, the interactions and imitation between traders leading to collective behaviour similar to crowd phenomena. Specifically, local interactions between traders may lead to the spontaneous bottom-up formation of clusters of agents of the same opinion. Different types of collective regimes are separated by so-called critical points which, in physics, are widely considered to be one of the most interesting properties of complex systems. A system goes critical when local influences propagate over long distances and the average state of the system becomes hyper-sensitive to a small perturbation, i.e., different parts of the system become highly correlated. Another characteristic is that critical systems are self-similar across scales: at the critical point, an ocean of traders who are mostly bearish may have within it several continents of traders who are mostly bullish, each of which in turns surrounds seas of bearish traders with islands of bullish traders; this progression continues all the way down to the smallest possible scale: a single trader [36]. Intuitively speaking, critical self-similarity is why local imitation cascades through the scales into global coordination (that is, from fine to coarse scale). Critical points are described in mathematical parlance as singularities associated with bifurcation and catastrophe theory. At critical points, scale invariance holds and its signature is the power-law behaviour of observables, for instance the price equation (1) is a power law as a function of time with exponent α. The origin of self-similarity is the critical point. The origin of the critical point is the occurrence of a delicate balance between order and disorder that may happen at certain times. Order is achieved through the forces of imitation and herding. Disorder reflects the idiosyncratic behaviours of agents reacting to private information.

The approach to the critical time signalling the end of the speculative bubble as well as the most probable time for a crash is analogous to the mechanism of ‘sweeping of an instability’ discussed in [25] for percolation and spin models (see also chapter 14 of [27]). According to this mechanism, the so-called control parameter (here, this is the time to the
end of the bubble) is not fixed but evolves dynamically as in the percolation model of stock market prices described in [33]. If, in addition, there is a positive feedback of the ‘order parameter’ (the price for instance, or the strength of imitation) onto the ‘control parameter’, the latter can be driven forcefully rather than in a random fashion towards its critical point value according to the mechanisms of positive feedback described in [24] and made concrete in the financial context in [29]. The mechanisms of positive feedback include rational and irrational herding, derivative hedging, portfolio insurance and so on. This corresponds to an imitative behaviour between agents strengthening progressively up to the critical point. Complementary to these cellular automata representations, a purely dynamical model in the spirit of [11] also describes the effect of the positive feedback mechanism in creating a finite-time singularity, which is the ‘critical point’. In [11], three mechanisms are combined to produce such finite-time singularities with approximately log-periodic oscillations preceding them. These mechanisms are (i) inertia (an analysis performed today based on past data affects the future with some delay), (ii) nonlinear decision-making by fundamental analysis leading to nonlinear reversal behaviour and (iii) nonlinear decision-making by technical strategies giving rise to positive feedback in trend-following prices. In particular, the interplay between inertia and nonlinear reversal behaviour produces oscillations with frequencies that are amplitude-dependent, a generic property of nonlinear oscillators. If the price increases, so does the frequency, leading to approximate log-periodicity. We stress that all these mechanisms are distinct from the concept of self-organized criticality (SOC) of [2], in that the critical point signalling the end of a bubble is not produced by a statistically stationary process about a trend. In contrast with the SOC state, here we are describing alternating convergences and divergences towards, and retreating from, a dynamical critical point (or a finite-time singularity) while the SOC concept describes systems that work continuously towards a dynamical critical point. This dynamical SOC is in general characterized by a power-law distribution of the sizes of the relaxation events (or ‘avalanches’). A given avalanche can, for instance, quantify the flux due to exchanges triggered by a given initiating sell.

The last ingredient of the model is to recognize that the stock market is made of actors which differ in size by many orders of magnitude, ranging from individuals to gigantic professional investors, such as pension funds. Furthermore, structures at even higher levels, such as currency influence spheres (US$, Euro, Yen ...), exist and with the current globalization and de-regulation of the market one may argue that structures on the largest possible scale, i.e. the world economy, are beginning to form. This means that the structure of the financial markets has features which resemble those of hierarchical systems with ‘traders’ on all levels of the market. Of course, this does not imply that any strict hierarchical structure of the stock market exists, but there are numerous examples of qualitatively hierarchical structures in society. Models of imitative interactions on hierarchical structures predict that the basic power-law behaviour can be enriched by so-called log-periodic corrections. Indeed, through the existence of preferred scales in a discrete hierarchy, or a discrete cascade of instabilities [26] or the existence of a competition between positive (trend-following) and negative (stemming from fundamental analysis) nonlinear feedbacks [11], the scale invariance characterizing critical points may be partially broken into a discrete scale invariance: that is, the observable is invariant with respect to changes of scale which are integer powers of a fundamental scaling ratio λ. It is easy to see that log-periodicity as given by the term $C_1 \tau^\alpha \cos[\omega \ln(\tau) + \phi_1]$ of expression (1) is the signature of discrete scale invariance: the term $\cos[\omega \ln(\tau) + \phi_1]$ reproduces itself each time $\ln(\tau)$ changes by $2\pi/\omega$, that is, each time $\tau$ is multiplied by $\lambda = \exp(2\pi/\omega)$. This theory of collective behaviour resulting from imitative nonlinear processes predicts robust and universal signatures of speculative phases of financial markets, both in accelerating bubbles as well as in decelerating antibubbles. These precursory patterns have been documented for essentially all crashes on developed as well as emergent stock markets, on currency markets, on company stocks, etc, as shown in [12–14, 17, 18, 21] and especially [28, 31].

2.2. Log-periodic fits

We use equations (1) and (3) to fit the logarithm of the S&P500 index over an interval starting at time $t_{\text{start}}$ and ending at 24 August 2002. The use of the logarithm is justified if one expects the price drop associated with the crash to be proportional to the present price [14]. In contrast, if the crash price drop is unrelated to the price, one should choose the price as the correct observable. The choice of $t_{\text{start}}$ is not completely obvious. Ideally, $t_{\text{start}}$ should be taken equal to $t_c$, the critical time, defining the end of the preceding bubble and the start of the antibubble. However, $t_c$ is not known a priori and should be determined self-consistently from the analysis of the data. It is clear that $t_{\text{start}}$ should be close to the peak of the S&P500 index in 2000 but cannot be expected to be exactly coincident with the time of the peak due to finite-size and other effects spoiling the validity of the log-periodic power law as well as the simultaneous end of the bubble and start of the antibubble.

We address this problem in two ways. First, we scan $t_{\text{start}}$ and select 10 time series, starting respectively at $t_{\text{start}} = 1$ March 2000, 1 April 2000, ..., 1 December 2000. The comparison of the fits obtained for these 10 time series will give a sense of their sensitivity with respect to $t_{\text{start}}$. Second, we notice that we can generalize the definition of $t_c$ given by (2) into

$$\tau = |t - t_c|.$$  (4)

While definition (2) together with power-law singularities associated with formulae (1) and (3) impose that $t_c < t_{\text{start}}$ for an antibubble, definition (4) allows for the critical time $t_c$ to lie anywhere within the time series. In that case, the part of the time series for $t < t_c$ corresponds to an accelerating ‘bubble’ phase while the part $t > t_c$ corresponds to a decelerating ‘antibubble’ phase. Definition (4) has thus the advantage of introducing a degree of flexibility in the search space for $t_c$ without much additional cost. In particular, it allows us to avoid a thorough scanning of $t_{\text{start}}$ since the value of $t_c$ obtained with this procedure is automatically adjusted without constraint.
There are two potential problems associated with this new procedure using (4). First, it assumes that the antibubble is always associated with a bubble which, in addition, has the same \( t_c \). Second, it assumes that the bubble and antibubble are exactly symmetric around \( t_c \), that is, the same parameters characterize the index evolution for \( t < t_c \) and for \( t > t_c \). For the cases relevant to this study, these two problems are quite minor and can be neglected because \( t_c \) is always found close to \( t_{\text{start}} \) (within the time series for \( t_{\text{start}} \) prior to August 2000 and anterior to the time series otherwise). Our comparison with fits using (2) shows that the new procedure provides significantly better and more stable fits, with in particular a value of \( t_c \) very weakly sensitive to \( t_{\text{start}} \). The parameters of the fits with the first-order formula using (2) or (4) are presented in Table 1, using subscripts 0 and 1 respectively. The fits with the first-order formula (1) with the definition (2) are unstable, are quite sensitive to \( t_{\text{start}} \) and have on average much larger values of the standard deviation of their residuals \( \chi \). Comparing parameters with subscripts 0 and 1, one can see that applying the symmetry condition (4) improves the quality of fit remarkably. We stress, however, that this improvement (4) is not crucial and our results reported below remain robust using definition (2). It is possible to relax the constraints of (4) by replacing \( \tau = \left| t - t_c \right| \) by an asymmetric function allowing in addition for a plateau or time lag. Since this would involve additional and possibly poorly constrained additional parameters, we do not pursue this possibility here.

The results of the fits of the logarithm of the S&P 500 index from \( t_{\text{start}} \) to 24 August 2002 with equations (1) and (3) using the improved scheme (4) are presented in Figure 2 and in Table 1 under the subscripts 1 and 2 respectively. The ten oscillating curves correspond to the ten best fits, one for each of the ten chosen values of \( t_{\text{start}} \) from March to December 2000. Over the approximately two year period available for the S&P500 antibubble, we find that the two formulae give essentially the same results and the same predictions for the following year. This is reflected quantitatively by the facts that \( \chi_1 > \chi_2 \) and that the parameters \( \Delta t \) are extremely large, in which case expression (3) reduces to equation (1) in the limit \( \Delta t \rightarrow +\infty \). The top (respectively bottom) panel of figure 2 corresponds to using equation (1) (respectively (3)). The curves are shown as continuous lines in their fitting interval and as dotted lines in their extrapolation to the future. Note the very robust nature of the solutions obtained for the ten choices of \( t_{\text{start}} \), which essentially all agree in their parameters and in their prediction of the future evolution.

To sum up, varying the starting date \( t_{\text{start}} \) of the fitted time window over a 10-month period and using two alternative formulae, we confirm that a single log-periodic power law describes very well the S&P500 antibubble since around mid-2000. According to the values of \( t_{\text{start}} \) listed in Table 1, the critical \( t_c \) is around 9 August 2000. This is consistent with the fact that the residuals of the fit obtained with \( t_{\text{start}} = 1 \) March 2000, 1 April 2000, 1 October 2000, 1 November 2000 and 1 December 2000 are significantly larger (\( \chi_1 > 3.3 \)) than the residuals for the other values of \( t_{\text{start}} \). It is natural that fits with \( t_{\text{start}} \) close to \( t_c \) have smaller fit residuals. The fits with \( t_{\text{start}} = 1 \) May 2000, 1 June 2000, 1 July 2000, 1 August 2000 and 1 September 2000 indeed exhibit smaller residual error. This conclusion is also supported by the numerical values of...
The S&P500 index antibubble fitted from $t_{\text{start}}$ to 24 August 2002, with the improved scheme (4) inserted in the two formulae (1) (upper panel) and (3) (lower panel) for different choices of $t_{\text{start}}$, spanning from 1 March 2000 to 1 December 2000. The dotted curves show the predicted future trajectories. One can see that the fits are robust with respect to different starting dates.

\[ \omega \text{ and of } \alpha \text{ which are basically constant for these five starting dates. Combining all the information shown in table 1, we have the critical time } t_c = 9 \text{ August 2000 } \pm 5 \text{ days, the angular log-frequency } \omega = 10.30 \pm 0.17 \text{ and the exponent } \alpha = 0.69 \pm 0.02. \text{ The fits with the second-order formula (3) give similar results.}

The preferred value for the critical time $t_c = 9 \text{ August 2000 } \pm 5 \text{ days corresponds to a starting date for the antibubble which is several months later than the peak of the S&P500 index in March 2000.}$ This delayed behaviour is different from what occurred 11 years earlier for the Japanese Nikkei index for which the antibubble started immediately after the maximum of the index [15]. Since a speculative bubble (respectively antibubble) corresponds to a growing agent imitation in a market oriented positively (respectively negatively), their corresponding critical times have no reason to coincide exactly. It is reasonable to expect that there may, in some cases, but not systematically, be a phase where the market dynamics is more stochastic with no clear imitation process occurring. In other words, our interpretation is that the delayed start of the antibubble of the S&P500 until August 2000 describes the time needed for the generation of negative market sentiment and the beginning of their epidemic imitative spread.

2.3. Analysis using the Shank transformation on a hierarchy of characteristic times

The fundamental idea behind the appearance of log-periodicity is the existence of a hierarchy of characteristic scales, as explained in the previous sections. Conversely, any log-periodic pattern implies the existence of a hierarchy of characteristic timescales. This hierarchy of timescales is determined by the local positive maxima or minima of a function such as $\ln[p(t)]$. For the S&P500 index antibubble, let us consider the times $t_n$ at which the S&P500 index reached a local minimum. As seen in figure 2, there is a clear sequence of sharp minima. We number them from the most recent one $t_1 = 23$ July 2002, $t_2 = 21$ September 2001, $t_3 = 4$ April 2001, $t_4 = 20$ December 2000 up to the earliest one at $t_5 = 12$ October 2000, which is still obvious. According to the prediction of log-periodicity, the spacing between successive values of $t_n$ approaches zero as a geometric series as $n$ becomes large and $t_n$ converges to $t_c$. We have $t_1 - t_2 = 305 \text{ days}, t_2 - t_3 = 170 \text{ days}, t_3 - t_4 = 105 \text{ days and } t_4 - t_5 = 69 \text{ days.}$ Such an analysis has been previously performed in a few specific cases [5, 28, 35].

Specifically, log-periodicity predicts that the times $t_n$ are organized in a geometric time series such that

$$t_n - t_c = \frac{\tau}{\lambda^n},$$

where $\tau$ sets the time unit and

$$\lambda = \exp \left( \frac{2\pi}{\omega} \right)$$

is the preferred scaling ratio. The relation (5) leads to

$$\frac{t_n - t_c}{t_{n+1} - t_c} = \frac{t_n - t_{n+1}}{t_{n+1} - t_{n+2}} = \lambda,$$

which is a signature of the discrete self-similarity of the log-periodic oscillations. Using the previously determined dates $t_1, \ldots, t_5$, we obtain

$$t_1 - t_2 = 1.79,$$

$$t_2 - t_3 = 1.62,$$

$$t_3 - t_4 = 1.52.$$
From three successive observed values of $t_n$, say $t_n$, $t_{n+1}$ and $t_{n+2}$, we can obtain an estimation of the critical time by the following formula

$$t_c = \frac{t_{n+1}^2 - t_{n+2}t_n}{2t_{n+1} - t_{n+2} - t_n}.$$  \hspace{1cm} (11)

This relation applies the Shank transformation [3] to accelerate the convergence of series. In the case of an exact geometrical series, three terms are enough to converge exactly to the asymptotic value $t_c$. Notice that this relation is invariant with respect to an arbitrary translation in time. Applying (11) with $t_1$, $t_2$, $t_3$ gives $t_c = 2$ September 2000. Applying (11) with $t_2$, $t_3$, $t_4$ gives $t_c = 3$ July 2000. Applying (11) with $t_3$, $t_4$, $t_5$ gives $t_c = 1$ June 2000. These back-predictions for $t_c$ are compatible with the value of $t_c = 9$ August 2000 ±5 days given in table 1 determined by the log-periodic fits.

In addition, the geometric structure of the time series $t_1$, $t_2$, ... is such that the previous time, $t_{n+3}$, can be obtained from the first three by

$$t_{n+3} = \frac{t_{n+1}^2 + t_{n+2}^2 - t_n(t_{n+2} + t_{n+2})}{t_{n+1} - t_n}.$$  \hspace{1cm} (12)

Since time is measured backward as $n$ increases, we are interested in the time $t_0$ at which the next future minimum occurs. For this, we use formula (12) and put $n = 0$ to get

$$t_0 = \frac{t_1^2 + t_2^2 - t_1t_2}{t_2 - t_1} \approx 21$$ January 2004. \hspace{1cm} (13)

While this Shank analysis has the advantage of simplicity and of having an obvious geometrical interpretation, its weakness lies in using a geometric set of characteristic times $\{t_n\}$ whose identification may be quite subjective. In the present case, the minima are so distinctive and sharp that there is no ambiguity. But, the use of only three characteristic dates makes more serious the sensitivity to noise and is of course bound to ambiguitv. But, the use of only three characteristic dates makes more serious the sensitivity to noise and is of course bound to ambiguity. While the two previous analyses are suggestive, the parametric nature of the first one and the limited power of the second one require additional tests of the reported log-periodicity. With this goal, we now turn to objective approaches for the detection of log-periodicity by applying a spectral Lomb analysis [22]. The Lomb analysis is a spectral analysis designed for unevenly sampled data which gives the same results as the standard Fourier spectral analysis for evenly spaced data. We apply this spectral analysis to two types of signals. Following [12], the first one is obtained by detrending the logarithm of the S&P500 index, using the power law (without log-periodicity) with the exponent $\alpha$ determined from the previous fits with formula (1). The time series of the residuals of the simple power-law fit should be a pure cosine of $\ln r$ if log-periodicity was perfect. We also use a recently developed non-parametric approach called the $(H,q)$-analysis, which has been successfully applied to financial crashes [40] and critical ruptures [39] for the detection of log-periodic components.

**2.4. Spectral analysis**

While the two previous analyses are suggestive, the parametric nature of the first one and the limited power of the second one require additional tests of the reported log-periodicity. With this goal, we now turn to objective approaches for the detection of log-periodicity by applying a spectral Lomb analysis [22]. The Lomb analysis is a spectral analysis designed for unevenly sampled data which gives the same results as the standard Fourier spectral analysis for evenly spaced data. We apply this spectral analysis to two types of signals. Following [12], the first one is obtained by detrending the logarithm of the S&P500 index, using the power law (without log-periodicity) with the exponent $\alpha$ determined from the previous fits with formula (1). The time series of the residuals of the simple power-law fit should be a pure cosine of $\ln r$ if log-periodicity was perfect. We also use a recently developed non-parametric approach called the $(H,q)$-analysis, which has been successfully applied to financial crashes [40] and critical ruptures [39] for the detection of log-periodic components.

**2.4.1. Parametric detrending approach.** We first construct the following detrended quantity which is defined using equation (1):

$$r(t) = \ln \left( A_1 - \ln p(t) \right) - \left\langle \ln \left( A_1 - \ln p(t) \right) \right\rangle,$$ \hspace{1cm} (14)

where the bracket $\langle \rangle$ refers to the sample average. Table 1 shows that the values of $A_1$ and $\alpha$ are quite stable and approximately equal to 7.33 and 0.69 for different $t_{\text{fit}}$. In order to investigate the impact of the different choice of $t_c$, we first construct $\ln p(t) - A_1$ as a function of $\ln(t-t_c)$ for different $t_c$ by fixing $A_1 = 7.33$. We then detrend it directly by determining the exponent $\alpha$ from a linear fit, whose residuals $r(t)$ define the time series to be analysed for log-periodicity.

Figure 3 shows the residual time series $r(t)$ as a function of $\ln(t - t_c)$ for three different critical times $t_c$: 15 July 2000 (top panel), 1 August 2000 (middle panel) and 15 August 2000 (bottom panel). The log-periodic undulations are clearly visible. We then perform the spectral Lomb analysis on these residuals. The three corresponding Lomb periodograms shown in figure 4 are very consistent with each other. They all exhibit an extremely strong and significant peak close to the log-frequency $f = \omega/2\pi = 1.7$ with an amplitude larger than 120. Its harmonic shown as the downward pointing arrow is significant. As we shall discuss later in section 2.5, the existence of harmonics has been shown to be an important factor for qualifying log-periodicity [38,41]. The presence of this harmonic is thus taken as a confirmation of the presence of log-periodicity. The third peak at $f = 2.5 - 3$ is also significant but less well-constrained. The inset of figure 4 magnifies the Lomb periodogram in the neighbourhood of the largest peaks. The two highest Lomb peaks, for $t_c = 1$ August 2000 and $t_c = 15$ August 2000, are significantly higher than the highest peak for $t_c = 15$ July 2000, confirming that the critical time is somewhere between 1 August 2000 and 15 August 2000 (recall that section 2.2 found $t_c \approx 9$ August 2000 ±5 days). As the
number of data points used for the Lomb analysis is the same in all analyses throughout this paper, this warrants a comparison.

To further assess the sensitivity with respect to the choice of $t_c$, we choose 21 different $t_c$ evenly spaced in the time interval from 15 July 2000 to 13 September 2000. For each $t_c$, we perform the same analysis as above and obtain the highest Lomb peak and its associated log-frequency. The results are shown in figure 5. The log-frequency is found to decrease with $t_c$. This is due to the fact that, moving the critical time forward (respectively backward), the other end of the time series will decelerate (respectively accelerate) the log-periodic oscillations. Figure 5 confirms that the best critical time $t_c$ falls somewhere in the first half of August 2000. The corresponding log-frequencies are $f = 1.63–1.72$ (ω = 10.2–10.7), in agreement with table 1.

It is interesting that the most relevant angular log-frequency $\omega \approx 10$ fitted on the S&P500 index is found very close to twice the value ≈5 found previously for the Nikkei index and for many other market price time series, including emergent markets [18]. Actually, a small but noticeable peak at this value $f \approx 5/2\pi \approx 0.80$ can be seen in figure 4, strengthening the analogy quantitatively. It may thus be surmised that both the Nikkei and the S&P500 indices are characterized by a universal discrete hierarchy of (angular) log-periodic frequencies, all harmonics of a fundamental angular log-frequency close to 5. Other systems have previously exhibited the curious fact, also observed when comparing the S&P500 to the Nikkei indices, that the higher-order harmonics may have an amplitude larger than the fundamental value [20,38,41], as observed for the S&P500 index.

2.4.2. **Non-parametric $(H, q)$-analysis.** The $(H, q)$-analysis [39,40] is a generalization of the $q$-analysis [6,7], which is a natural tool for the description of discretely scale-invariant fractals. The $(H, q)$-derivative is defined as

$$D_q^H f(x) = \frac{f(x) - f(qx)}{\ln(1/q) x^H}.$$  (15)

The special case $H = 1$ recovers the normal $q$-derivative, which itself reduces to the normal derivative in the limit $q \to 1$. There is no loss of generality by constraining $q$ in the open interval $(0, 1)$ [39]. We apply the $(H, q)$-analysis to verify non-parametrically the existence of log-periodicity by taking $f(x) = \ln p(t)$ and $x = t - t_c$. The advantage of the $(H, q)$-analysis is that there is no need of detrending as done in section 2.4.1. Such detrending is automatically accounted for by the finite difference and the normalization by the denominator.

Figure 6 shows the $(H, q)$-derivatives of the logarithm of the S&P500 index as a function of $\ln(t - t_c)$ for three choices of $t_c$: 15 July 2000 with $H = 0$ and $q = 0.8$ in the top panel, 2 August 2000 with $H = 0.5$ and $q = 0.7$ in the middle panel, and 16 August 2000 with $H = 0.4$ and $q = 0.7$ in the bottom panel. Each of these pairs of $(H, q)$ is the optimal pair [39,40] corresponding to the most significant peak among all Lomb periodograms for each $t_c$. The log-periodic undulations are clearly visible. The Lomb periodograms of the $(H, q)$-derivatives shown in figure 6 are presented in figure 7. The three highest Lomb peaks are even more significant than those in figure 4. The inset shows the magnified Lomb peaks. The highest Lomb peak is obtained for $t_c = 2$ August 2000, which corresponds to the log-frequency $f \approx 1.71$.

To test the robustness of the $(H, q)$-analysis for the detection of log-periodicity, we scan $H$ from −1 to 1 with spacing 0.1 and $q$ from 0.1 to 0.9 with spacing 0.1 for each...
critical \(t_c\). Figure 8 shows the log-frequency \(f\) as a function of \(H\) and \(q\) for \(t_c = 16\) August 2000. The existence of a flat plateau at \(f = 1.62 \pm 0.07\) for most of the pairs \((H, q)\) confirms the existence of log-periodicity. The five smaller log-frequencies below the plateau correspond to the spurious values stemming from the most probable effect of noise on power laws [10] and should be discarded. The three higher log-frequencies above the plateau probably stem from the interaction between high-frequency noise and the second harmonics.

We then apply the non-parametric \((H, q)\)-analysis to the S&P500 index for 21 different choices of the critical \(t_c\). For each given \(t_c\), we take the average of the Lomb periodograms for all \(21 \times 9\) pairs of \((H, q)\). The amplitude of the highest peak in each averaged Lomb periodogram is plotted as a function of \(t_c\) in the lower panel of figure 9. Their associated log-frequency shown in the upper panel of figure 9 with the error bars estimated from the average height of the plateaux such as the one seen in figure 8. This log-frequency slightly decreases with \(t_c\). There are two clear humps in the lower panel around late July and mid-August 2000. The hump around mid-August is higher than the other one. The corresponding log-frequencies \(f \approx 1.60–1.70\) are compatible with those reported in table 1.
2.5. Role of log-periodic harmonics

The spectral Lomb analyses reported above (see figures 4 and 7) as well as the visual structure of the S&P500 time series suggest the presence of a rather strong harmonic at the angular log-frequency $2\omega$. The possible importance of harmonics in order to qualify log-periodicity is also made more credible by recent analyses of log-periodicity in hydrodynamic turbulence data [38, 41] which have demonstrated the important role of higher harmonics in the detection of log-periodicity.

We thus revisit the parametric log-periodic fits of section 2.2 with formula (1) using (4) to include the effect of a harmonic at the angular log-frequency $2\omega$. With this goal, we postulate the formula

$$\ln p(t) \approx A + B e^{\omega t} + C e^{\sigma t} \cos(\omega \ln(t) + \phi_1) + D e^{\phi_2} \cos(2\omega \ln(t) + \phi_2),$$

(16)

which differs from equation (1) by the addition of the last term proportional to the amplitude $D$. This formula has two additional parameters compared with (1), the amplitude $D$ of the harmonic and its phase $\phi_2$. We follow the fitting procedure of [42] with minor modifications, which itself adapts the slaving method of [12, 13]. By rewriting equation (16) as

$$\ln p(t) = A + B f(t) + C g(t) + D h(t),$$

where

$$f(t) = e^{\omega t}, \quad g(t) = g(t), \quad h(t) = h(t),$$

Solving analytically this system allows us to slave the four parameters $A, B, C$ and $D$ using the method of [12, 13]. By rewriting equation (16) as

$$\ln p(t) = A + B f(t) + C g(t) + D h(t),$$

we have

$$\sum_{i=1}^{N} \frac{\ln p_i}{(\ln p_i) f_i} \left( \sum_{i=1}^{N} \frac{\ln p_i}{(\ln p_i) h_i} \right) \left( \sum_{i=1}^{N} \frac{\ln p_i}{(\ln p_i) g_i} \right) \left( \sum_{i=1}^{N} \frac{\ln p_i}{(\ln p_i) h_i} \right) = \left( \begin{array}{cccc} A & B & C & D \\ f_1 & f_2 & f_3 & f_4 \\ g_1 & g_2 & g_3 & g_4 \\ h_1 & h_2 & h_3 & h_4 \end{array} \right),$$

(17)

where

$$p_i = p(t_i), \quad f_i = f(t_i), \quad g_i = g(t_i) \quad \text{and} \quad h_i = h(t_i).$$

Solving analytically this system allows us to slave the four parameters $A, B, C$ and $D$ to the other parameters in the search for the best fit. With this approach, we find that the search of the optimal parameters is very stable and provides fits of very good quality in spite of the remaining five free parameters.

The results are listed in table 2 and depicted in figure 10. The fit residuals are reduced considerably compared with the fits reported in table 1. The improvement of the fits is obvious when comparing figure 10 with figure 2. Note also that $|D| < |C| (|C| \approx 2 - 3|D|)$ which is in agreement with the fact that the spectral peak of the fundamental log-frequency is much higher than the peak of the second harmonic approximately by a factor of 5, as shown in figures 4 and 7.

We have also tested whether the addition of a third log-frequency around $\omega \approx 2.8$ (which is not a third harmonic), as suggested from the spectral Lomb analyses shown in figures 4 and 7, could improve and/or modify the fit. We found a slight but non-significant reduction of the root-mean-square residue $\chi$ with negligible modification of the fit, suggesting that this log-frequency is due to noise.

Since expression (16) contains the formula (1) as a special case $D = 0$, we can use Wilk’s theorem [23] and the statistical methodology of nested hypotheses to assess whether the hypothesis that $D = 0$ can be rejected. Therefore, the null hypothesis and its alternative are

$$H_0: D = 0; \quad H_1: D \neq 0.$$
Table 2. Parameters of the fits with equation (16) with (4) of the S&P500 index for different \( t_{\text{start}} \). The fit residuals are strongly reduced compared with the fits shown in table 1. The bold columns correspond to the values of \( t_{\text{start}} \) giving basically the same values for \( \omega \), \( \alpha \) and \( \phi \).

<table>
<thead>
<tr>
<th>( t_{\text{start}} )</th>
<th>03/01</th>
<th>04/01</th>
<th>05/01</th>
<th>06/01</th>
<th>07/01</th>
<th>08/01</th>
<th>09/01</th>
<th>10/01</th>
<th>11/01</th>
<th>12/01</th>
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<tr>
<td>( t )</td>
<td>06/15</td>
<td>07/15</td>
<td>07/16</td>
<td>08/04</td>
<td>08/12</td>
<td>08/21</td>
<td>08/04</td>
<td>06/14</td>
<td>08/10</td>
<td>06/04</td>
</tr>
<tr>
<td>( \chi ) × 100</td>
<td>3.009</td>
<td>2.858</td>
<td>2.713</td>
<td>2.711</td>
<td>2.680</td>
<td>2.690</td>
<td>2.661</td>
<td>2.700</td>
<td>2.712</td>
<td>2.687</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.79</td>
<td>0.68</td>
<td>0.65</td>
<td>0.65</td>
<td>0.61</td>
<td>0.61</td>
<td>0.54</td>
<td>0.60</td>
<td>0.57</td>
<td>0.57</td>
</tr>
<tr>
<td>( \omega )</td>
<td>12.12</td>
<td>11.56</td>
<td>11.77</td>
<td>10.70</td>
<td>10.74</td>
<td>10.74</td>
<td>10.26</td>
<td>10.97</td>
<td>12.24</td>
<td>10.86</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>3.88</td>
<td>5.01</td>
<td>3.76</td>
<td>1.41</td>
<td>1.39</td>
<td>4.52</td>
<td>2.89</td>
<td>6.28</td>
<td>0.59</td>
<td>1.35</td>
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<td>( \phi_2 )</td>
<td>4.90</td>
<td>0.66</td>
<td>4.39</td>
<td>2.98</td>
<td>2.80</td>
<td>2.84</td>
<td>5.83</td>
<td>0.24</td>
<td>1.20</td>
<td>2.88</td>
</tr>
<tr>
<td>( A )</td>
<td>7.33</td>
<td>7.34</td>
<td>7.34</td>
<td>7.34</td>
<td>7.34</td>
<td>7.33</td>
<td>7.38</td>
<td>7.39</td>
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<td>( B ) × 1000</td>
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<td>-5.59</td>
<td>-5.77</td>
<td>-7.60</td>
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<td>-12.42</td>
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<td>-10.34</td>
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<tr>
<td>( C ) × 1000</td>
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<td>-0.92</td>
<td>-1.09</td>
<td>1.21</td>
<td>1.54</td>
<td>1.58</td>
<td>-2.29</td>
<td>-1.44</td>
<td>2.03</td>
<td>1.79</td>
</tr>
<tr>
<td>( D ) × 1000</td>
<td>0.22</td>
<td>0.47</td>
<td>0.59</td>
<td>-0.53</td>
<td>-0.77</td>
<td>-0.71</td>
<td>-1.14</td>
<td>-0.77</td>
<td>-1.03</td>
<td>-0.96</td>
</tr>
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\( D = 0 \) is sufficient to explain the data, and favours the fit with \( D \neq 0 \) as statistically significant.

In our test, the beginning of the fitted data set is fixed at 9 August 2000, while the end of the data set varies from 1 January 2001 to 24 August 2002. The results of the Wilk test are presented in table 3. Increasing the number of points decreases the probability that the obtained probability to overpass \( T \) may result from chance, and thus increases the statistical significance of the fit with equation (16). Since the assumption of Gaussian noise is most probably an under-estimation of the real distribution of noise amplitudes, the very significant improvement in the quality of the fit brought by the use of equation (16) quantified in table 3 provides most probably a lower bound for the statistical significance of the hypothesis that \( D \) should be chosen non-zero, above the 99.8% confidence level. Indeed, a non-Gaussian noise with a fat-tailed distribution would be expected to decrease the relevance of competing formulas, whose performance could be scrambled and be made fuzzy. The clear and strong result of the Wilk test with assumed Gaussian noises thus confirms a very strong significance of equation (16).

Strengthened by this analysis of the strong relevance of the second harmonics at \( 2\omega \), we revisit the Nikkei index and fit it in the first 2.6 years of its decay starting in January 1990 using (16) to test whether the second harmonics are also important for the Nikkei index. The fit is compared with the log-price in figure 11. We find indeed an impressive improvement, as the root-mean-square residue \( \chi = 0.0457 \) is significantly smaller than the root-mean-square residue \( \chi = 0.0535 \) obtained with expression (1) and whose fit is shown in figure 1.

3. Discussion and prediction

Starting from a visual analogy with the Nikkei index shifted by 11 years, the first point of the analyses presented above is to have established with strong significance the existence of an antibubble followed by the S&P500 index approximately since July–August 2000. This antibubble is characterized by an overall power-law decay of the index decorated by strong log-periodic oscillations.

Following the analogy with the trajectory of the Nikkei index 11 years earlier, the second point is that a comparison between the fits obtained with equations (1) and (3) shows that the S&P500 index has not yet entered into the second phase in which the angular log-frequency may start its shift to another value, as did the Nikkei index after about 2.5 years of its decay. We may expect this to occur in the future. Not being able to estimate directly the parameter \( \Delta \), controlling this transition due to the smallness of the S&P500 index antibubble duration, we can however offer the following guess, based on the hypothesis that the values \( \Delta_t \) and \( \Delta_{\omega} \) given by the fit of expression (3) to the logarithm of the Nikkei index are reasonable estimates of those for the S&P500 index. We also use the parameters in the column of \( t_{\text{start}} = 1 \) August 2000 of table 1 for the first-order regime and extrapolate the fitted curve to 2006 (continuous curve). Using the values of the fits with (1) and plugging in the values of \( \Delta_t \) and \( \Delta_{\omega} \) from the Nikkei index in expression (3) gives the dashed curve shown in figure 12. The crossover from the first-order regime to the second-order regime is here suggested to occur in the first half of 2004. Figure 12 also compares the first-order fit and the second-order guess with formula (16) taking into account the second harmonics. Figure 12 suggests that the next broad minimum of the S&P500 index will occur in the first semester...
The US 2000–2002 market descent: how much longer and deeper?

Table 3. Likelihood-ratio test of the hypothesis that \( D \neq 0 \) in equation (16). The beginning of the data set for fit is set at 9 August 2000. The end of the data set varies from 1 January 2001 to 24 August 2002. The \( n \) column gives the number of data for each fit. The confidence level quantified by Proba decreases on average to 24 August 2002. The end of the data set varies from 1 January 2001 equation (16). The beginning of the data set for fit is set at

\[
\Delta t_{\text{last}} \quad n \quad n \sigma_1^2 \quad n \sigma_2^2 \quad T \quad \text{Proba} (\%)
\]

<table>
<thead>
<tr>
<th>Date</th>
<th>( n )</th>
<th>( n \sigma_1^2 )</th>
<th>( n \sigma_2^2 )</th>
<th>( T )</th>
<th>Proba (%)</th>
</tr>
</thead>
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<td>0.0283</td>
<td>18.7</td>
<td>0.0015</td>
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<td>0.0381</td>
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</tr>
<tr>
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<td>0.0440</td>
<td>36.2</td>
<td>&lt;10(^{-4})</td>
</tr>
<tr>
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<td>0.0616</td>
<td>10.3</td>
<td>0.13</td>
</tr>
<tr>
<td>05/01/01</td>
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<tr>
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<tr>
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<td>01/26/02</td>
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<tr>
<td>02/25/02</td>
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<td>04/26/02</td>
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<td>05/26/02</td>
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</tbody>
</table>

of 2004. This is consistent with the prediction (13) using Shank’s transformation.

The third important point is the improvement in the quality of the fits and therefore in the potential for predictions when adding the harmonics at \( 2\omega \), as shown in figure 10 compared with figure 2. Figure 2 suggests a local maximum of the S&P500 index around the end of the first quarter of 2003, while figure 10 refines this prediction by seeing an earlier peak before but close to the end of 2002. These two predictions are not in contradiction: the prediction of figure 10 shows that the oscillatory structure of the S&P500 index implies several ups and downs in the coming year, with a tendency to appreciate for a while before going down again by the end of 2003.

Ideally, we would like to combine the effect of the second-order formula (3) with the impact of the second harmonics described by expression (16). We do this by adding to (16) a term with the same structure as the one proportional to \( C_2 \) in (3) with \( \omega \) replaced by \( 2\omega \), again fixing \( \Delta_1 \) and \( \Delta_2 \) at the values determined from the Nikkei index. Figure 13 compares the result of this fit with that with (16) and their extrapolation up to close to the end of 2006. These two curves provide a sense of the future directions of the S&P500 index and their probable degree of variability.

4. Concluding remarks

The growing awareness in 2002 of the crisis in the American financial system is reminiscent of the starting point of Japan’s massive financial bubble burst more than 10 years before and of the intertwining of the bad debts and bad performance of banks whose capital is invested in the shares of other banks, thus creating the potential for a catastrophic cascade of bankrupts. Japan has rediscovered before the US the faults of the 19th century financial system in the US in which stock markets were so much intertwined with their overall banking financial system, that busts and bursts occurred more than once every decade, with firms losing their credit lines and workers and...
consumers their savings and often their employment. It is often said that the 1930s depression was the last of the stock market and bank-induced economic collapses. The growing fuzziness between financial banking systems and stock markets, in part due to the innovations in information technology, has re-created the climate for stronger bubbles and more pronounced losses of confidence leading to long-lived bearish regimes possibly nucleating depressions.

A big problem is that, in the collapse following them, policy interventions such as lowering interest rates, reducing taxes, government spending packages and any measure to restore investors’ confidence may be much less effective, as discovered with the Japanese so-called liquidity trap, a process in which government and the central bank policy becomes essentially useless. In addition, loss of confidence by investors (for instance following the frauds in accounting in the US) may lead to a non-negligible cost to the overall economy [8], providing a positive feedback reinforcing the bearish climate.

We have proposed that the trajectories of the US and Japanese stock markets could be understood in large part by taking into account imitative and herding mechanisms, both stemming possibly from rational or irrational behaviour. A key ingredient entering probably in the imitative and herding processes is the phenomenon of investor confidence. It has recently been argued [34] that investor confidence can be understood far better if one assumes not that investors have rational expectations, but that they have what economists call ‘adaptive expectations’. Individuals with rational expectations predict others’ behaviour by focusing on their external incentives and constraints. In contrast, individuals with adaptive expectations predict others’ behaviour (including possibly the behaviour of such an abstract ‘other’ as the stock market) by extrapolating from the past. In addition, confidence and trust in the market have been shown to be subject to historical effects [34]. We believe that these behavioural traits provide fundamental roots underlying the validity of our analysis which, ultimately, can be viewed similarly as nothing but a (rather complex nonlinear) extrapolation of the past.

Our theory does not describe the common ‘stationary’ evolution of the stock market, but rather is specifically tailored for identifying ‘monsters’ or anomalies (bubbles and their end) and for classifying their agones. We claim that these agones (the antibubbles) are mostly shaped by collective effects between economic and stock market agents, with their imitation and confidence (and lack thereof) idiosyncracies. Ultimately, our description must leave place to a recovery of the fundamental pricing principles (and of possible emergences of new speculative bubbles) but, before this, it describes the way in which collective effects control in large part the processes towards this recovery.

This study complements and makes more precise a previous one focusing on longer times scales, based on three pieces of evidence, namely the growth over long timescales of population, gross national product and stock market indices [19, 28], which issued a prediction that starting around 1999, a 5–10 year consolidation of international stock markets will occur, allowing a purge after the over-aggressive appetite of the preceding decade. For more than the last two years, this prediction has been borne out. The present study confirms this impression that the US stock market is not yet on the verge of recovery.

With its extraordinary and unparalleled growth, its ensuing decade-long absence of growth, its crowded land and its aging population, is Japan a precursor of the new regime that mankind has to shift to, as discussed in [19, 28]? It seems that the present qualification of the US market to be in an antibubble phase is entirely in line with these predictions. From a larger perspective and at the horizon of the end of the first half of this century, the behaviour of these stock markets raises the following question: shall we learn the lessons of previous bubbles and crashes/depressions and shall we be able to transit to a qualitatively different organization of economic and cultural exchanges before the fundamental limitations of a finite earth and limited human intelligence set in?

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References

Arthur W B 1996 Increasing returns and the new world of business Harvard Business Rev. 74 100–9
Krugman P R 1996 The Self-Organizing Economy (Cambridge, MA: Blackwell)
Irwin S H and Yoshimaru S 1999 Managed futures, positive feedback trading, and futures price volatility J. Futures Markets 19 759–76
Watanabe T 2002 Margin requirements, positive feedback trading, and stock return autocorrelation: the case of Japan Appl. Financial Economics 12 395–403

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