Short term synaptic depression on the E-I connection within a balanced network creates bistability

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Introduction

The status quo
Classical models of balanced networks are only capable of producing one fixed firing regime. These models cannot accurately model certain networks in the brain that are capable of transitioning from one steady state firing regime to another. Thus, it is necessary to develop a biophysically-principled model that has multiple steady state solutions.

Balanced networks
A balanced network has stronger synaptic strength parameters than a traditional Wilson-Cowan network. The stronger synapses allow for internally generated variability as population size approaches infinity. A balanced network is also an inhibition-stabilized network, a common biophysical feature in many areas of the cortex.

Order of Synaptic Connection Strength
(N = population size)

Wilson-Cowan	Balanced
$O(1/N)$	$O(1/\sqrt{N})$

Synapses

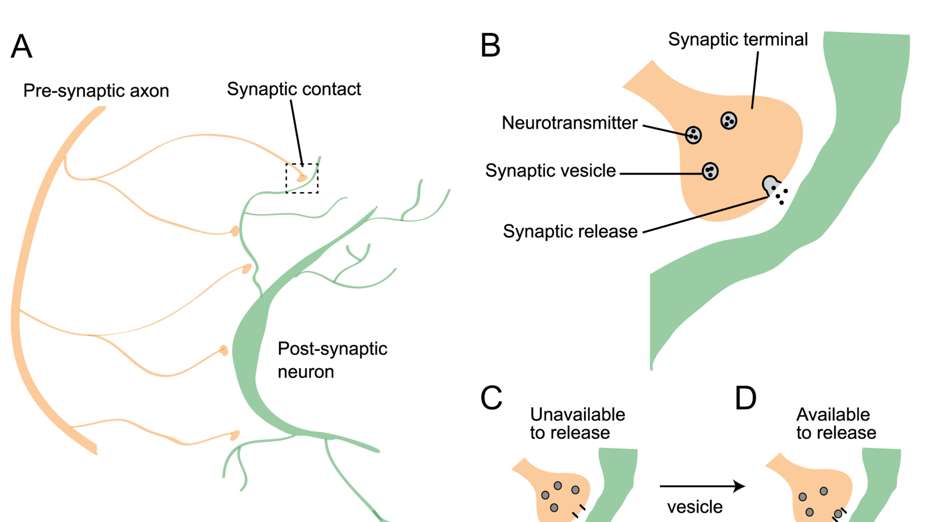
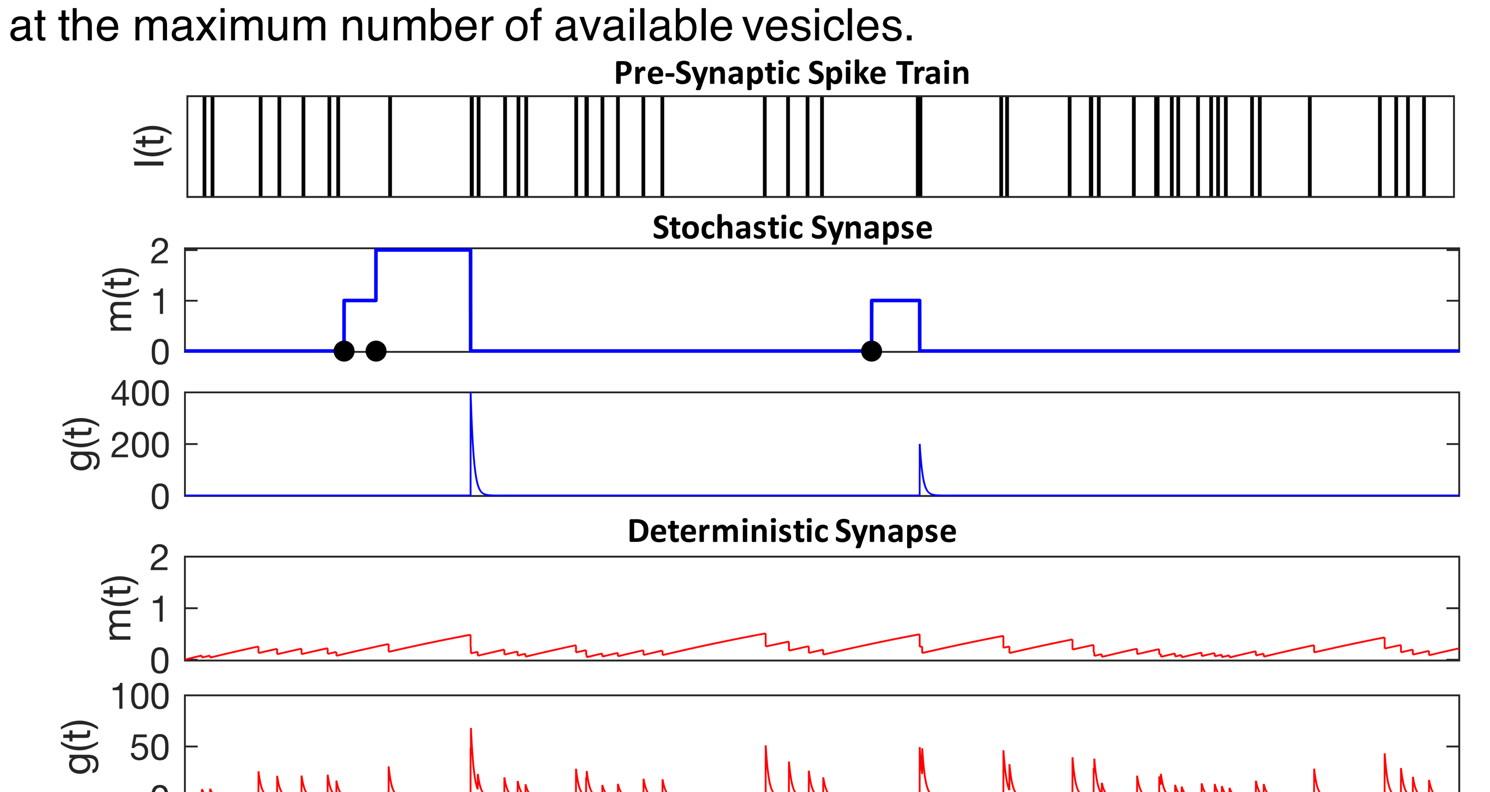


Figure 1 A-D, Rosenbaum et al. 2012

Short Term Synaptic Depression
In our model, the activation of vesicles in the pre-synaptic neuron increases the conductance of the post-synaptic neuron. When vesicles are activated the number of available vesicles at a synapse decreases. This process of depression and consequent recovery of vesicles takes place on a timescale of a few hundred milliseconds, therefore we set our time constant of vesicle recovery τ_u accordingly.

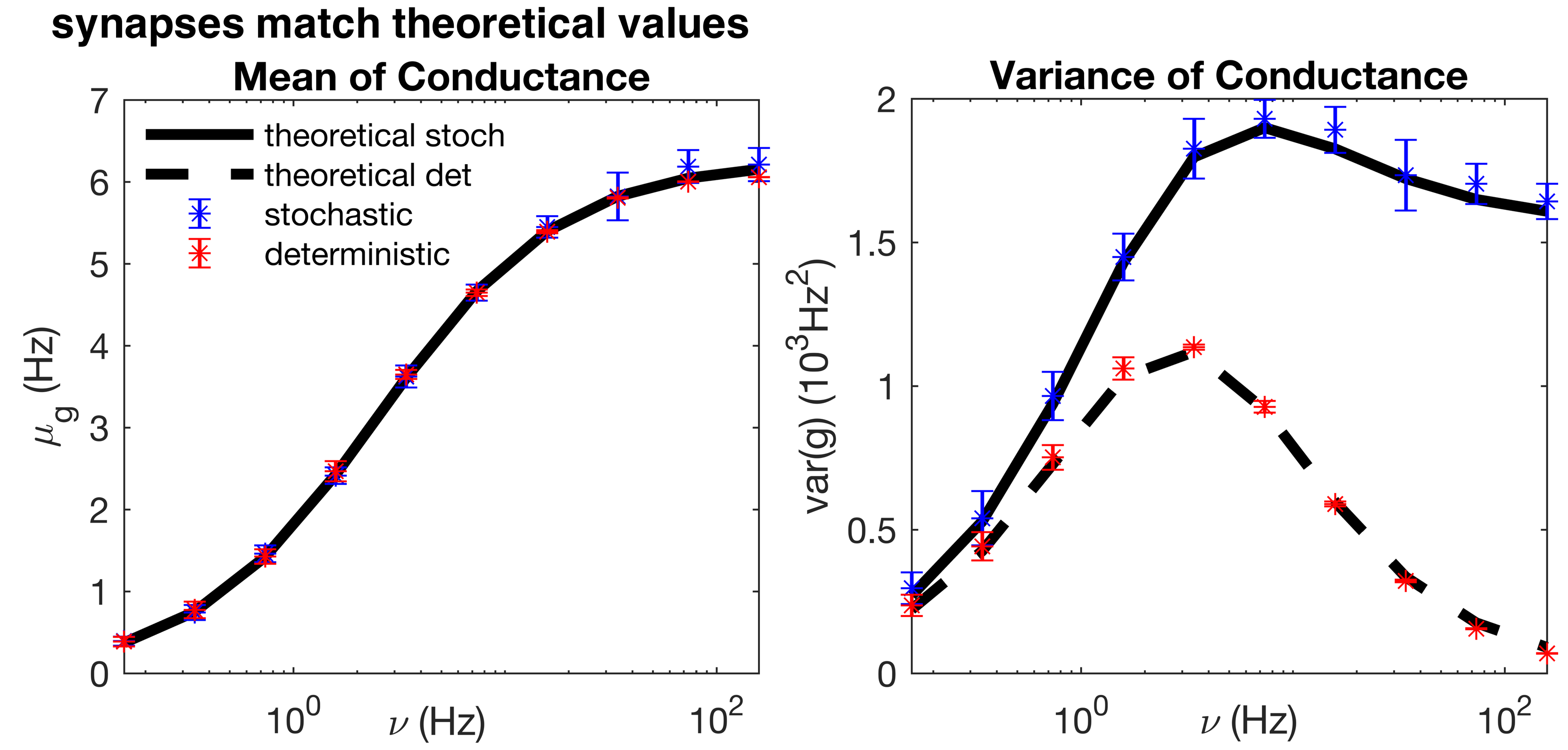
Stochastic vs. Deterministic Synaptic Transmission
Synaptic transmission is inherently stochastic. However, when trial averaging, a deterministic model suffices as a mean field approximation of a stochastic synapse. Each vesicle in a stochastic synapse has an independent probability p_r of being activated when a spike comes in. In a deterministic synapse, a spike removes a fraction p_r of the available vesicles. Recovery of synaptic resources also differs. In a stochastic model, recovery events (black dots) are independent for each vesicle and are determined by a Poisson process with parameter $\lambda = 1/\tau_u$. In a deterministic model, synaptic resources increase with an asymptote at the maximum number of available vesicles.



Adapted from Figure 2 A-D, Rosenbaum et al. 2012
m(t) – Available vesicles/synaptic resources
g(t) – Conductance of post-synaptic cell

Single Depressing Synapse

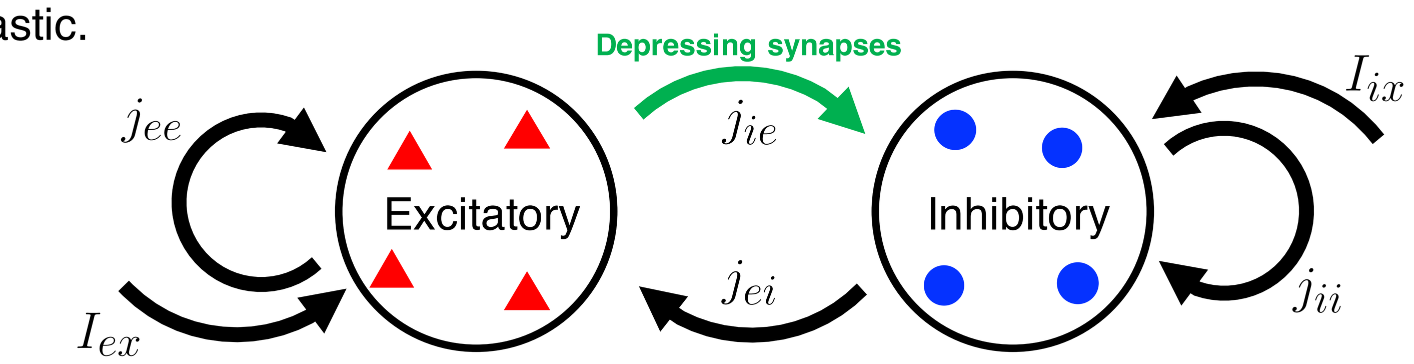
Conductance statistics from simulations of stochastic and deterministic synapses match theoretical values



The mean conductance is the same for both stochastic and deterministic models, but at high pre-synaptic firing rates the variance in conductance of the deterministic model approaches zero.

Network Model

Network Schematic
Synaptic depression is added to the E→I connection. All other synapses are not plastic.



Self Consistent Firing Rate Solution
To find the theoretical firing rates of the excitatory and inhibitory populations we use the following self consistency relationship:

$$r_\alpha = \tau_m\sqrt{\pi}\int_{(\mu_\alpha - V_t)/\sigma_\alpha}^{(\mu_\alpha - V_r)/\sigma_\alpha} e^{z^2} erfc(z)dz$$

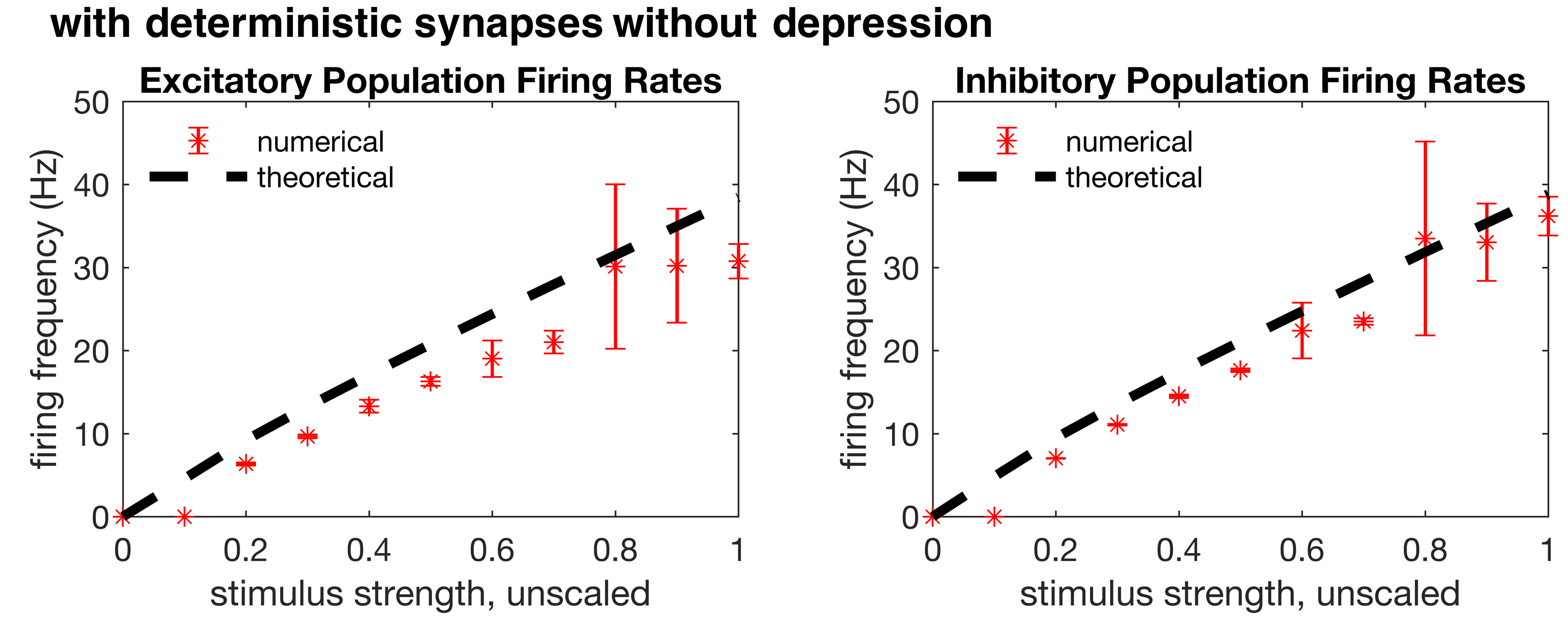
$$\mu_\alpha = j_{\alpha e}K_e r_e \tau_m + j_{\alpha i}K_i r_i \tau_m + I_{\alpha x}$$

$$\sigma_\alpha^2 = j_{\alpha e}^2 K_e r_e \tau_m + j_{\alpha i}^2 K_i r_i \tau_m + \sigma_{b_\alpha}^2$$

$$\alpha \in \{e, i\}$$

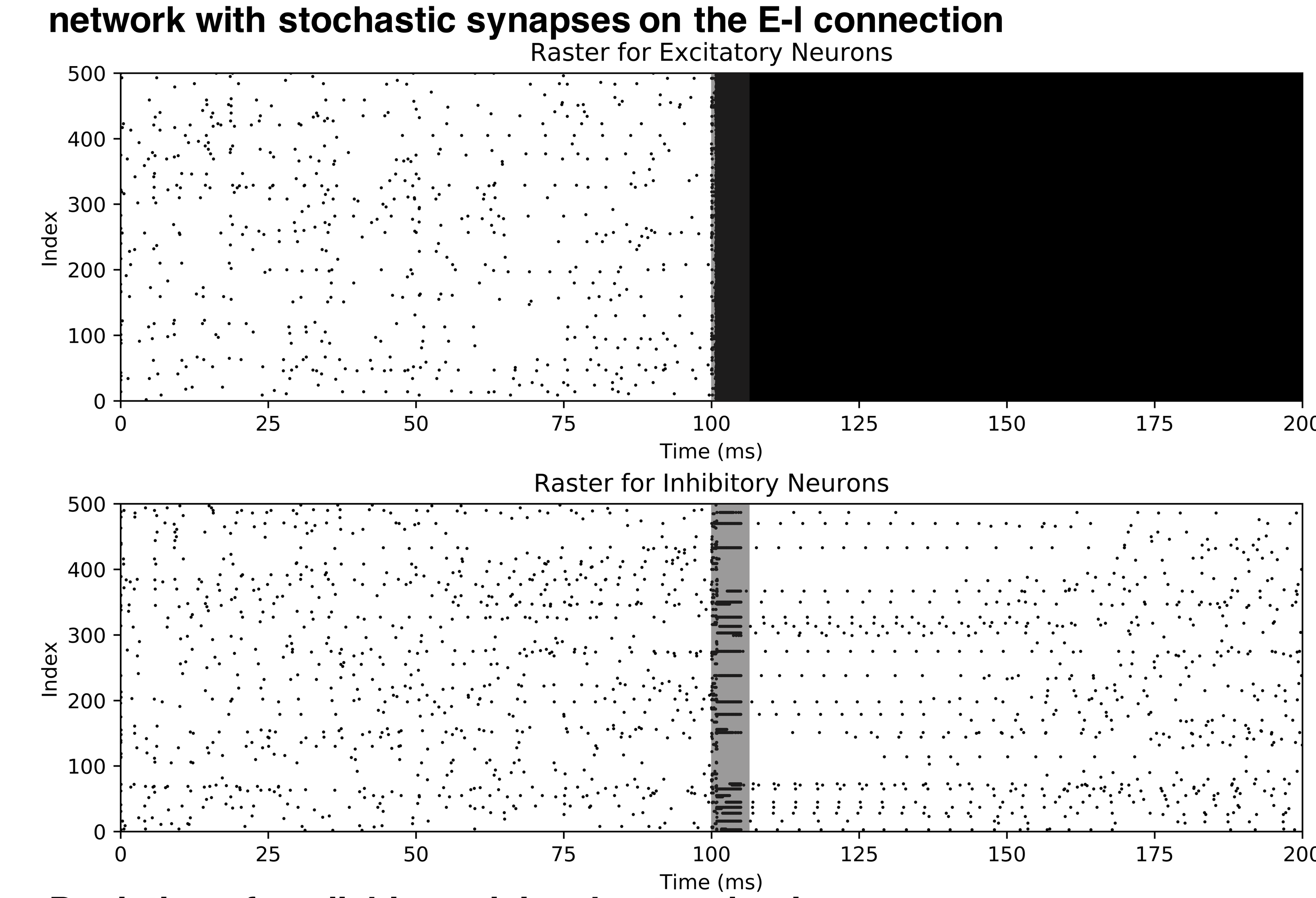
Name	Definition
r_α	firing rate of population α
τ_m	membrane time constant
V_t	firing threshold
V_r	reset potential
μ_α	mean input to population α
σ_α	standard deviation of input to population α
$j_{\alpha\beta}$	strength of connection from a neuron in population β to a neuron in population α
K_α	expected number of connections from population α
$I_{\alpha x}$	external input to population α
p_r	probability of vesicle release
M	maximum number of possible available vesicles
τ_u	time constant of vesicle recovery
σ_{b_α}	background noise to population α

Theoretical firing rates are similar to simulated firing rates for network with deterministic synapses without depression

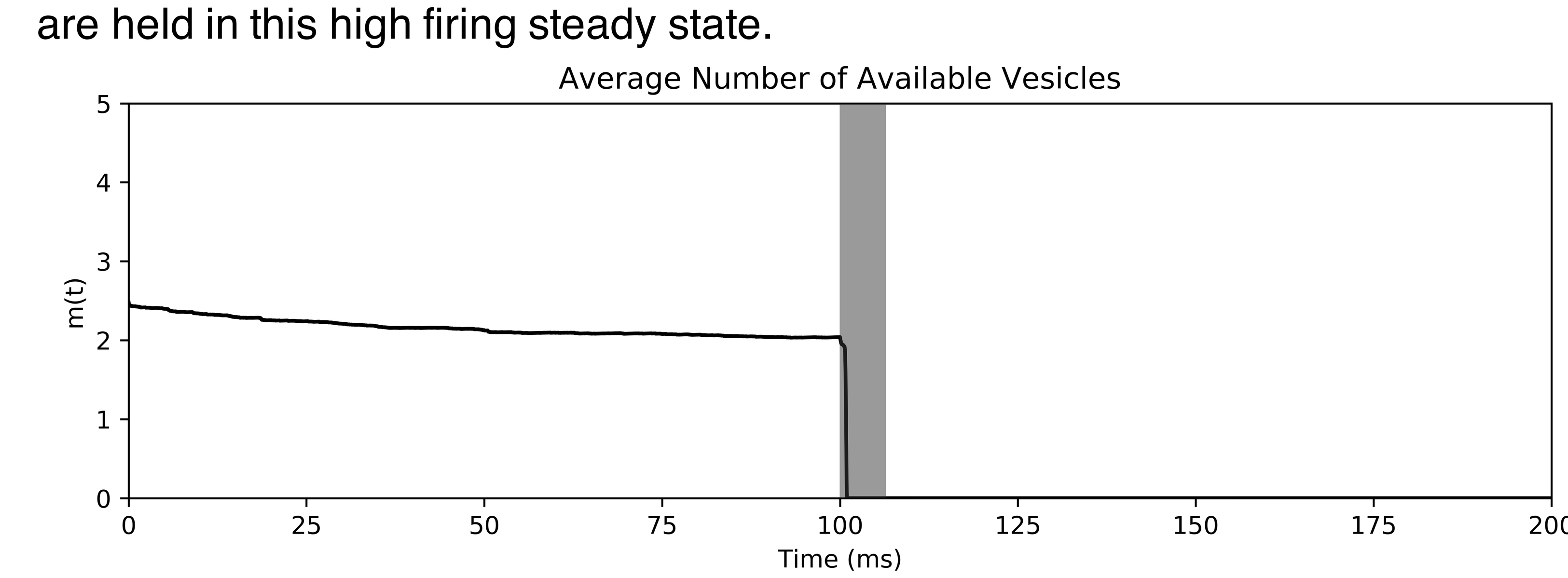


Bistability

Transitions are possible between steady state firing regimes in a balanced network with stochastic synapses on the E-I connection



Depletion of available vesicles due to stimulus
When the stimulus arrives it causes all neurons to fire at very high rates (excitatory avg. 85kHz, inhibitory avg. 2.4kHz). This severely depletes the synaptic resources of the E-I connection. When the stimulus ends, the neurons are held in this high firing steady state.



Discussion

Conclusions
The theoretical firing rates calculated with the self consistency relationship nearly approximate the simulated firing rates of a network with deterministic synaptic depression.

The addition of synapses with stochastic depletion onto the E-I connection of a balanced network creates multiple fixed point solutions for excitatory and inhibitory population firing rates. However, the observed firing rates were physiologically unreasonable.

Future Directions
To find parameters for which the network can transition from one realistic firing rate to another we can test various parameter sets in the self-consistency equation used to find theoretical firing rates. Alternatively, we could incorporate a refractory period, which would force the neuron to fire below a certain rate.

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