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Introduction

- Numerous causal learning models have been proposed to explain how individuals make inferences about the relationship between a cue and a subsequent effect
- **Goal:** Determine which causal learning models best predict subjects' responses while examining model consistency within subject across conditions

Data

Two experimental datasets from Danks & Schwartz (2005, 2006) were analyzed:

- Participants were presented with a sequence of binary cause-effect cases
- After each case, the participant estimates the strength of the relationship between the cause and effect on a scale from -100 to 100
- Sequences are non-stationary: at the halfway point in each trial, the causal relationship switches direction

Procedure

Generate Predictions

Calculate SSE

Find Best Fit Model

A grid search was employed to generate predictions for reasonable sets of parameter values for each model

For each participant in each condition, the sum of squared errors between the participant data and the model predictions was found

The causal learning model with the set of parameters that generated the lowest SSE is declared the model of best fit fo that participant in that condition

Individual Variation in Causal Learning

	Ca	ausal Le	earnir
		Exis	sting Mo
	Augmented Rescorla-Wag	ner ΔV	$T_i = lpha_{i\gamma(C_i)}$
	Proportion of Confirming ΔV_j^{i+1} = Instances (PCI)		
	Catena Belief Adjustment $J_i =$		
	Causal Suppor	t	support =
	Power PC	$\Delta V_i = lpha_{i\gamma(i)}eta_{\delta(E)}$	$(\lambda\delta(E)-\prod_{\delta}$
	Sequential Bayesian Theo	ory $P(ec{w}_{t-})$	$_{+1} D_t, m) = \ _{+1} D_{t+1}, m)$
	Conditional Probabilistic C	ontrast	$\Delta P_i = P$
		Nc	ovel Moc
	Bayesian Corre Optimization	elation $Pr($	$\rho W) = \frac{Pr}{M}$
	Bayesian Corre Optimization Moving V k-Window	elation $Pr($ = $\frac{1}{k}(N(C, E))$	$egin{aligned} & W) = rac{Pr}{-M} & W = r$
	Bayesian Corre Optimization Moving V k-Window Win-Stay, Lose Shift	elation $Pr($ $= rac{1}{k}(N(C,E)+$ $V_{i+1} =$	$egin{aligned} & \rho W) = rac{P \eta}{-1} \ & + N (\neg C, \neg I) \ & \int V_i + rac{1 - V_i}{2} \ &5 \ & V_i - rac{1 + V_i}{2} \ & .5 \end{aligned}$
	Bayesian Corre Optimization Moving V k-Window Win-Stay, Lose Shift Rescorla-Wag with decay	elation $Pr($ $= rac{1}{k}(N(C,E)+$ $V_{i+1} =$ ner	$egin{aligned} & ho W) = rac{Pr}{-M} & ho W) = rac{Pr}{-M} & ho W & ho V_i + N(eggin{aligned} & ho V_i + rac{1-V_i}{2} & ho V_i + rac{1-V_i}{2} & ho V_i + rac{1+V_i}{2} & ho V_i + rac{1+V_i}$
	Bayesian Corre Optimization Moving V k-Window Win-Stay, Lose Shift Rescorla-Wage with decay Rescorla-Wage with stability of	elation $Pr($ $= \frac{1}{k}(N(C, E) + V_{i+1}) + V_{i+1} =$ ner	$egin{aligned} & W) = rac{Pi}{-M} \ &+ N(\neg C, \neg D) \ &iggle V_i + rac{1-V_i}{2} \5 \ V_i - rac{1+V_i}{2} \ .5 \ &\Delta V_i = \mu lpha_i \ & ext{where } \mu = V_i \ &\Delta V_i = \mu lpha_i \ & ext{where } \mu = V_i \ & ext{where$
	Bayesian Corre Optimization Moving V k-Window Win-Stay, Lose Shift Rescorla-Wage with decay Rescorla-Wage with stability of Power PC with decay	elation $Pr($ $= \frac{1}{k}(N(C, E) + V_{i+1})$ ner of beliefs $\Delta V_i = \mu \alpha_{i\gamma(C_i)}/V_{i+1}$ where $\mu = e^{-\gamma * r}$	$egin{aligned} & W) = rac{P_{II}}{-1} \ &+ N(eggregan C, eggregan L, end{aligned} &+ N(eggregan C, eggregan L, end{aligned} &+ N(eggregan C, eggregan L, end{aligned} &+ N(eggregan C, eggregan L, eggregan L, end{aligned} &+ N(eggregan L, eggregan L, eggregan L, end{aligned} &+ N(eggregan L, eggr$

ng Models

odels

$$(\lambda_{i}) eta_{\delta(E)} (\lambda \delta(E) - \sum_{\delta(C_j)} V_j)$$

$$=eta((-1)^{|\delta(E)-\delta(C_j)|}\lambda-V_j^i)$$

 $J_{i-1} + \gamma(PCI - J_{i-1})$ $= \log\left(\frac{P(D|\text{Graph 1})}{P(T)}\right)$

 $_{\delta(V_k)=1}(1-V_k)[1-\prod_{\delta(V_i)=1}(1-V_j)])$

 $=\int P(ec{w}_{t+1}ec{w}_t)P(ec{w}_tert D_t,m)dec{w}_tec{w}_tec{w}_t$ $= P(D_{t+1}|ec{w}_{t+1},m)*P(ec{w}_{t+1}|D_t,m)$ $P(D_{t+1}|D_t)$

$$P(E|i\cap Q)-P(E|
eg i\cap Q)$$

dels

$$egin{aligned} ⪻(W|
ho)Pr(
ho)\ Pr(W) \end{aligned}$$
 where $W = X_1 + X_2$
 $(Pr(W)) &= M(V(\neg C, E) + N(C, \neg E)) \end{aligned}$
 $V_i \geq 0, \delta(E) = \delta(C)$
 $V_i \geq 0, \delta(E) \neq \delta(C)$
 $V_i < 0, \delta(E) \neq \delta(C)$
 $V_i < 0, \delta(E) = \delta(C)$
 $u_{i\gamma(C_i)} eta_{\delta(E)} (\lambda \delta(E) - \sum_{\delta(C_j)} V_j)$
 $&= e^{-\gamma n}$
 $u_{i\gamma(C_i)} eta_{\delta(E)} (\lambda \delta(E) - \sum_{\delta(C_j)} V_j)$
 $&= \max(\gamma \sigma, \frac{1}{\sqrt{n}})$
 $&= \prod_{\delta(C_k)=1} (1 - V_k) [1 - \prod_{\delta(C_j)} (1 - V_j)])$



- Bayesian models
- predictions

Conclusions

• Overall, the expanded causal power theories exceeded all other causal learning models in prediction accuracy

• The Sequential Bayesian Theory performed better on unbiased sequences, plausibly due to the order invariance property of

 Both the Catena Belief Adjustment and PCI models generated improved predictions on shorter sequences, suggesting that these models capture the volatility of early causal

• Future research directions include an analysis into model consistency within subject and the overlap in the prediction space of various models