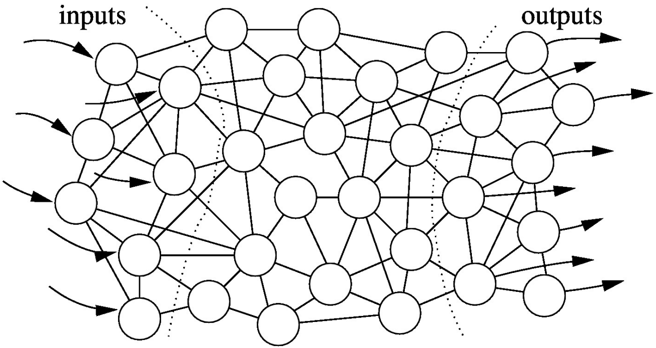
Constraint satisfaction



- Units represent **hypotheses** about parts of a problem
- Weights code **constraints** on how hypotheses can combine (i.e., the degree to which they are consistent or inconsistent)
- Possible **solutions** correspond to particular patterns of active units
- External input introduces **bias** to favor one possible solution over others

Example: Map coloring

Assign colors to regions so that no adjacent regions have the same color.

Hypotheses (units)

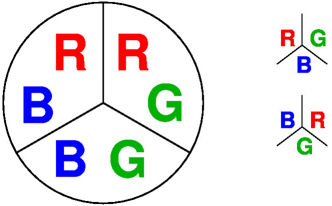
- An assignment of a color to a region

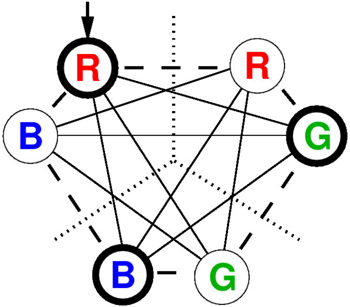
Constraints (weights)

- Adjacent regions must be assigned different colors
  (negative between-regions weights for matching colors)
- Only one color can be assigned to each region
  (negative within-region weights for non-matching colors)
- Each region must be assigned a color
  (positive between-region weights for non-matching colors)

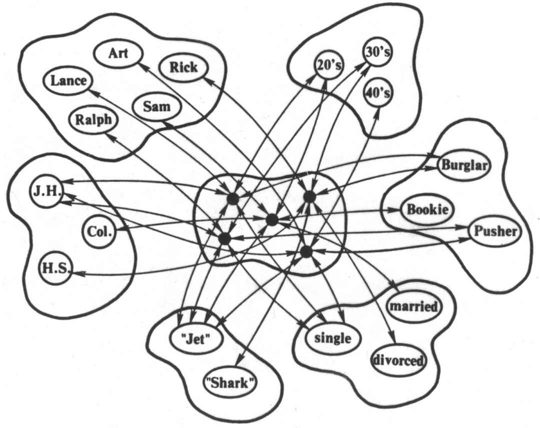
Biases

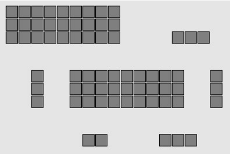
- Initial color preference for a given region





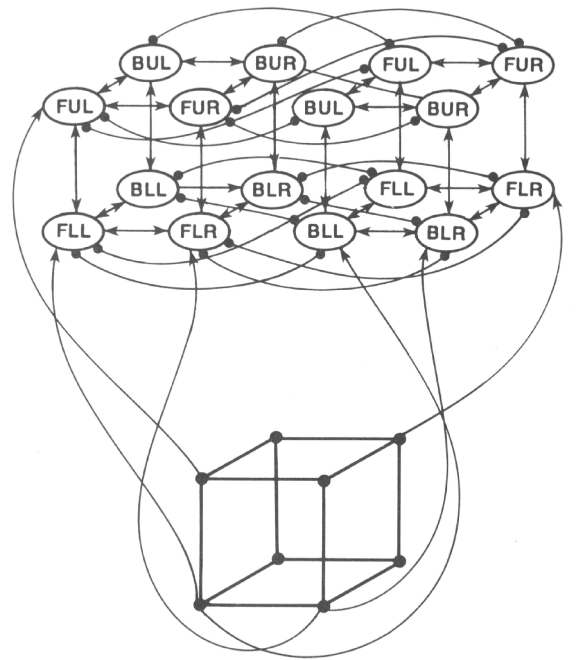
Jets and Sharks (McClelland, 1981)





Name	Gang	Age	Edu	Mar	Occupation
Art	Jets	40's	J.H.	Sing.	Pusher
Al	Jets	30's	J.H.	Mar.	Burglar
Sam	Jets	20's	COL.	Sing.	Bookie
Clyde	Jets	40's	J.H.	Sing.	Bookie
Mike	Jets	30's	J.H.	Sing.	Bookie
Jim	Jets	20's	J.H.	Div.	Burglar
Greg	Jets	20's	H.S.	Mar.	Pusher
John	Jets	20's	J.H.	Mar.	Burglar
Doug	Jets	30's	H.S.	Sing.	Bookie
Lance	Jets	20's	J.H.	Mar.	Burglar
George	Jets	20's	J.H.	Div.	Burglar
Pete	Jets	20's	H.S.	Sing.	Bookie
Fred	Jets	20's	H.S.	Sing.	Pusher
Gene	Jets	20's	COL.	Sing.	Pusher
Ralph	Jets	30's	J.H.	Sing.	Pusher
Phil	Sharks	30's	COL.	Mar.	Pusher
Ike	Sharks	30's	J.H.	Sing.	Bookie
Nick	Sharks	30's	H.S.	Sing.	Pusher
Don	Sharks	30's	COL.	Mar.	Burglar
Ned	Sharks	30's	COL.	Mar.	Bookie
Karl	Sharks	40's	H.S.	Mar.	Bookie
Ken	Sharks	20's	H.S.	Sing.	Burglar
Earl	Sharks	40's	H.S.	Mar.	Burglar
Rick	Sharks	30's	H.S.	Div.	Burglar
Ol	Sharks	30's	COL.	Mar.	Pusher
Neal	Sharks	30's	H.S.	Sing.	Bookie
Dave	Sharks	30's	H.S.	Div.	Pusher

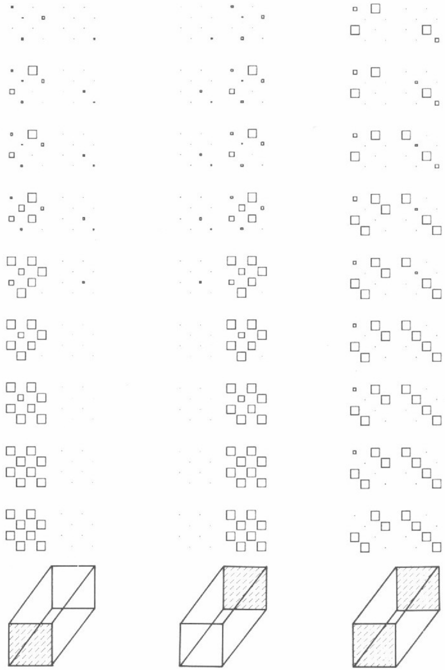
Necker Cube



Necker cube settling

- Start with small random activations
- Network typically settles into stable state in which either the left or right subnetwork is fully active (first two columns on right)
  - coherent (3D) interpretations
- Occasionally network settles into mixed state with some units on in each subnetwork (third column)
  - incoherent (2D?) interpretation

⇒ Why should we think that a network will settle to a **good** interpretation?



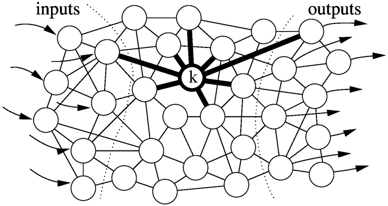
Maximizing Goodness (= minimizing Energy)

Global measure of degree to which activations satisfy weight constraints

$$G\,(\text{Goodness} = -\text{Energy}) = \sum_{i<j} a_i\, a_j\, w_{ij}$$

How should binary unit k behave locally so as to increase global Goodness?

- Set $a_k = 1$ if $G_{a_k=1} > G_{a_k=0}$ (or, equivalently, $G_{a_k=1} - G_{a_k=0} > 0$)



$$\begin{aligned} G_{a_k=1} &= \sum_i a_i\, w_{ik} + \sum_{i<j\neq k} a_i\, a_j\, w_{ij} \\ G_{a_k=0} &= \sum_{i<j\neq k} a_i\, a_j\, w_{ij} \\ G_{a_k=1} - G_{a_k=0} &= \sum_i a_i\, w_{ik} \end{aligned}$$

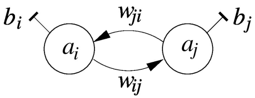
- Set $a_k = 1$ if $\sum_i a_i\, w_{ik} > 0$ (= **binary threshold unit**)

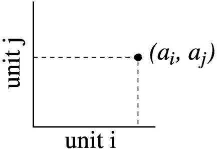
Sigmoid units: increase activation as net input increases (monotonicity)

Goodness surface in state space

State space

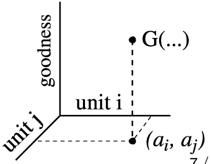
- A high-dimensional space with a dimension for each of *n* units in the network
- Each unit's activity (state) can be interpreted as a coordinate along its corresponding axis/dimension
- At any instant in time, the current pattern of activity over the entire network corresponds to a particular *n*-dimensional point in the space
- As units update their states, the point moves in state space



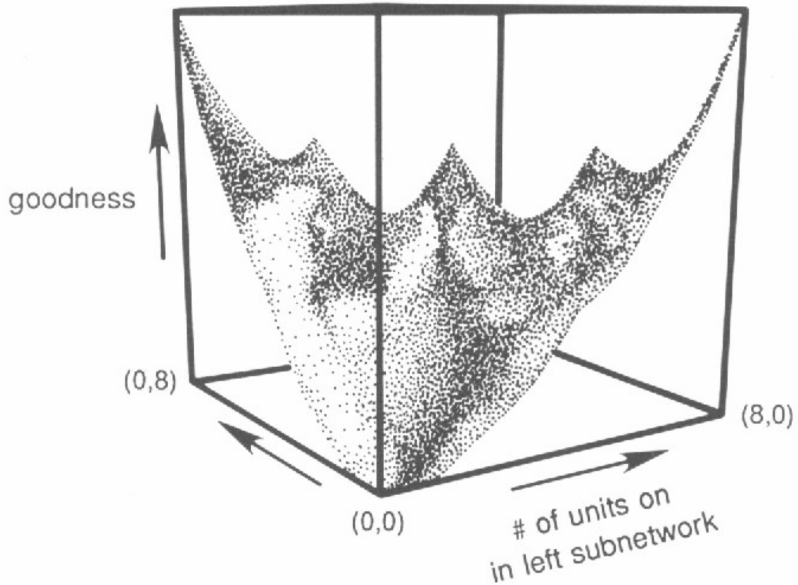


Goodness surface

- Each instantaneous pattern of activity has a corresponding "goodness" value
- Add an additional (n+1st) dimension to state space so that the goodness of each point (activity pattern) can be plotted "above" it
- The set of goodness values corresponding to all possible activity patterns forms a continuous surface "above" state space



Goodness surface (Necker cube)



Goodness surface with input to front face (left cube)