

CIS 630 assignment #7

Due: Nov. 23 at class meeting.

Problem 1 (20')

We say two variables x, y are *conditionally independent* given z , if

$$p(x, y|z) = p(x|z)p(y|z). \quad (1)$$

This corresponds to a simple causal graph:

$$x \leftarrow z \rightarrow y$$

Prove that this is equivalent to the statement

$$p(x|y, z) = p(x|z), \quad \text{or} \quad p(y|x, z) = p(y|z) \quad (2)$$

That is, you can derive (1) from (2), and derive (2) from (1).

Problem 2 (30')

You are a witness of a night-time hit-and-run accident involving a taxi at Athens. All taxis in Athens is either green or blue. You swear under oath that the taxi was blue. Extensive testing shows that under dim lighting conditions, discrimination between blue and green is 75% reliable. Is it possible to calculate the most likely color for the taxi? (hint, distinguish carefully between the proposition that the taxi *is* blue, and the proposition that it *appears* blue (your belief). Suppose you are a jury, formulate the problem, and compute the probability of the car was blue given the testimony? Now what if you have another piece of information that 9 out of 10 taxis in Athens are green? Will that change the jury's belief? Re-do the computation.

Problem 3 (50')

For the Burglary-earthquake problem, we have the causal network and five probability tables (in the book or blackboard discussion). Compute the following probabilities

1. $p(B)$ — the probability (your belief) for a Burglary without evidence.
2. $p(B|A)$ — what is your belief of the Burglary if you hear the alarm directly?
3. $p(B|J)$ — what is your belief of the Burglary if John reported to you about the alarm?
4. $p(B|M)$ — what is your belief of the Burglary if Mary reported to you about the alarm?
5. $p(B|M, J)$ — what is your belief of the Burglary if Mary and John both reported to you about the alarm?

For each probability, you need to express it in terms of probabilities in the 5 tables. Then calculate the number. Try to discuss whether your calculated numbers make sense. (hint: you may not save computation by remember some intermediate results)